

Formleg mál og reiknanleiki

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skil 4

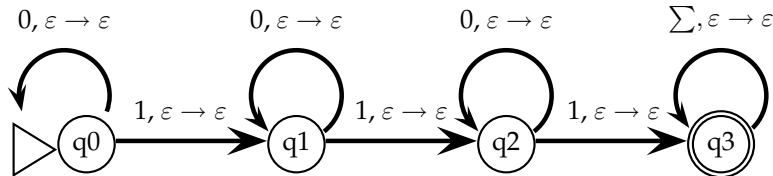
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1 Exercise 2.5 bls 120

Give informat descriptions and state diagrams of pushdown automata. In all parts the alphabet Σ is $\{0,1\}$.

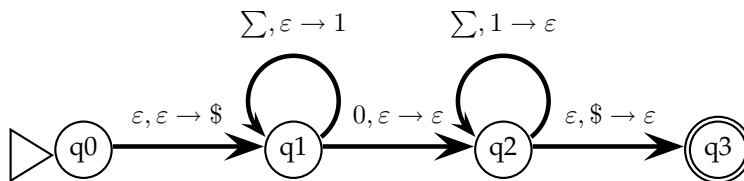
a) $\{w \mid w \text{ contains at least three 1s}\}$

Hérna er okkur alveg sama um 0, við verðum með 4 state. (0) er bara byrjunin, (1) er kominn einn 1, (2) er kominn einn 2, (3) er kominn einn 3. Þegar við erum kominn í state 3 þá er okkur alveg sama hvað kemur og state 3 er einnig lokastaða.



d) $\{w \mid w \text{ the length of } w \text{ is odd and its middle symbol is a 0}\}$

Við setjum \$ fyrst á staflann setjum svo 1 á staflann í hver skipti sem kemur einn. Ef við sjáum 0 þá meigum við fara í næstu stöðu og þegar við sjáum staf þá tökum við 1 af staflanum. Þegar við sjáum bara \$ á staflanum og bandið er búið þá hættum við og samþykkjum strenginn.



2 Exercise 2.6 bls 120

Give context free grammars generating the following languages.

a) The set of strings over alphabet {a,b} with twice as many a's as b's

$$S \Rightarrow S_1 a S_1 a S_1 b S_1 \mid S_1 a S_1 b S_1 a S_1 \mid S_1 b S_1 a S_1 a S_1$$

$$S_1 \Rightarrow S_1 a S_1 a S_1 b S_1 \mid S_1 a S_1 b S_1 a S_1 \mid S_1 b S_1 a S_1 a S_1 \mid \varepsilon$$

b) The complement of the language $\{a^n b^n \mid n \geq 0\}$.

$$\text{allt annað en } S \Rightarrow a S b \mid \varepsilon$$

$$S \Rightarrow S_1 \mid S_2 \mid S_3$$

$$S_1 \Rightarrow a S_1 a \mid b S_1 b \mid a \mid b \mid a S_1 b \mid b S_1 a \mid a a \mid b b \mid b a$$

$$S_2 \Rightarrow a \mid a S_2 \mid b S_2 S_2 \mid S_2 b S_2 \mid S_2 S_2 b$$

$$S_3 \Rightarrow b \mid b S_3 \mid a S_3 S_3 \mid S_3 a S_3 \mid S_3 S_3 a$$

3 Exercise 2.15 bls 121

Show that the class of context-free languages is closed under the regular operations, union, concatenation, and star

Context-free \geq regular

Union

$$G_1 = (V_1, \Sigma_1, R_1, S_1)$$

$$G_2 = (V_2, \Sigma_2, R_2, S_2)$$

$$G_0 = G_1 \cup G_2$$

$$G_0 = (V_1 \cup V_2 \cup \{S_0\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S_0 \Rightarrow S_1 \mid S_2\}, S_0)$$

Concatenation

$$G_1 = (V_1, \Sigma_1, R_1, S_1)$$

$$G_2 = (V_2, \Sigma_2, R_2, S_2)$$

$$G_0 = G_1 G_2$$

$$G_0 = (V_1 \cup V_2 \cup \{S_0\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S_0 \Rightarrow S_1 S_2\}, S_0)$$

Star

$$G_1 = (V_1, \Sigma_1, R_1, S_1)$$

$$G_0 = (G_1)^*$$

$$G_0 = (V_1 \cup \{S_0\}, \Sigma_1, R_1 \cup \{S_0 \Rightarrow S_1 S_0 \mid \varepsilon\}, S_0)$$