Reiknirit, rökfræði og reiknanleiki

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skil 13

1 Exercise 10.7 bls 378

Show that BBP $\subseteq$ PSPACE.

For a given input x, enumerate all random strings of polynomial size, and count how many of these strings are accepted by the BPP algorithm on the given input x. If more than half strings are accepted, then Accept; otherwise, Reject. Since both enumeration and counting an be done in polyspace, we have that BPP is in PSPACE. (There are only $2^{\text{poly}(n)}$ possible random strings of given polynomial size; this number can be written down using only poly(n) bits.)

2 Exercise 10.11 bls 378

Let M be a probabilstic polynomial time Turing machine and let C be a language where, for some fixed $0 < \varepsilon_1 < \varepsilon_2 < 1$,
A) $w \notin C$ implies Pr[M accept w] $\leq \varepsilon_1$, and
B) $w \in C$ implies Pr[M accept w] $\geq \varepsilon_2$

Let the algorithm A for L be such that, for $0 < \varepsilon_1 < \varepsilon_2 < 1$, if $x \in L$, then the algorithm accepts with probability at least ε_2, and if $x \notin L$, then the algorithm accepts with probability at most ε_1. There are three cases.

1) $\varepsilon_1 < \frac{1}{2}$ and $\varepsilon_2 > \frac{1}{2}$. Then the algorithm has error $\varepsilon \leq \max\{\varepsilon_1, 1 - \varepsilon_2\} < \frac{1}{2}$. By Lemma 10.5, we can make this error probability exponentially small.

2) $\frac{1}{2} \leq \varepsilon_1 < \varepsilon_2 < 1$. Then define $\delta = \frac{1}{(\varepsilon_1 + \varepsilon_2)}$. Consider the new algorithm B where, on input x, B flips a biased coin whose probability of "heads" is δ. If the coin falls "heads", then B simulates A, accepting iff A accepts. If the coin is "tails", then B Rejects. Note that, for $x \in L, B$ accepts x with probability $\delta\varepsilon_2$, and for $x \notin L, B$ accepts x with probability $\delta\varepsilon_1$. Also note that, by the choice of δ, we have $\delta\varepsilon_1 < \frac{1}{2}$ and $\delta\varepsilon_2 > \frac{1}{2}$. Thus, the new algorithm B satisfies case (1) above and B has the error probability $\delta\varepsilon_1 < \frac{1}{2}$. Thus, we can apply Lemma 10.5 to B and reduce the error probability to an exponentially small value.

3) $\varepsilon_1 < \varepsilon_2 < \frac{1}{2}$. In this case, by reversing the roles of Accept and Reject for the algorithm A, we obtain the algorithm A for the complement of L so that the new algorithm A satisfies case (2) above. Hence, we can reduce the error probability for recognizing the complement of L. Let us call the resulting algorithm C. By exchanging the roles of Accept and Reject in C, we obtain an algorithm for L, which still has an exponentially small error probability.

Thus, in all three cases, we conclude that $L \in$ BPP.