Reiknirit, rökfræði og reiknanleiki

Magni Þór Birgisson

skil 8

1 Exercise 5.18 bls 165

Let S be an r.e. set; prove that the sets $D = \bigcup_{x \in S} \text{dom}\phi_x$ and $R = \bigcup_{x \in S} \text{ran}\phi_x$ are both r.e.

domϕ	=	the domain of the partial function ϕ
ranϕ	=	the range of the partial function ϕ

2 Exercise 5.19 bls 165

Let K_t be the set $\{x | \exists y \leq t, \text{step}(x, x, y) > 0\}$; that is, K_t is the set of functions that converge on the diagonal in at most t steps.

1) Prove that, for each fixed t, K_t is recursive, and verify the equality $\bigcup_{t \in \aleph} K_t = K$.

2) Conclude that, if S is an r.e. set, the set $\bigcap_{x \in S}$ domϕ_x need not be r.e.

3 Exercise 6.1 bls 173

Reduce Exact Cover by Two-Sets to the general matching problem. An instance of the first problem is composed of a set containing an even number of elements, say 2N, and a collection of subsets of the set, each of which contains exactly two elements. The question is: "Does there exist a subcollection of N subsets that covers the set?" An instance of general matching is an underected graph and the objective is to select the largest subset of edges such tat no two selected edges share a vertex. Since the general matching problem is solvable in polynomial time and since your reduction should run in very low polynomial time, it follows that Exact Cover by Two-Sets is also solvable in polynomial time.

4 Exercise 6.2 bls 173

The decision version of the smallest subsets problem, known as K-th Largest Subset, is:"Given a set of objects with associated sizes and given integers B and K, are there at least K distinct subsets for which the sum of the size of elements is less the or equal to B?" Show how to reduce the partition problem to K-th Largest Subset. Excluding the work donw inside the procedure that solves instances of K-th largest subset problem, how much work is done by your reduction? (This is a reduction that requires a large number of subroutine calls: use binary search to determine the number of subsets for which the sum of the sizes of the elements is less than or equal to half the total sum.)