Reiknirit, rökfræði og reiknanleiki

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skil 6

1 Exercise 5.11 bls 164

Prove that the following functions are primitive recursive by giving a formal construction.

1) The fcuntion exp(n, m) is the exponent of the mth prime in the prime power decomposition of n, where we consider the 0th prime to be 2. (Forinstance we have exp(1960,2) = 1 because 1960 has a single factor of 5.)

$\exp(n, m) = \text{prime}(m)|n$

2) The function max $y \leq x[g(y, z_1, \ldots, z_n)]$, where g is primitive recursive, returns the largest value in $\{g(0, \ldots), g(1, \ldots), \ldots, g(x, \ldots)\}$.

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{otherwase} \end{cases}$$

$max(x, y) = \text{add}(\text{mult}(\text{less}(x, y), y), \text{mult}(\text{not}(\text{less}(x, y)), x))$
$f(0, z_1, \ldots, z_n) = g(0, z_1, \ldots, z_n)$
$f(i + 1, z_1, \ldots, z_n) = \max(P_2^{n+2}(i, f(i, z_1, \ldots, z_n), z_1, \ldots, z_n), g(i + 1, z_1, \ldots, z_n))$

3) The Fibonacci function $F(n)$ is defined by $F(0) = F(1) = 1$ and $F(n) = F(n-1)+F(n-2)$.(Hint: use the course-of-values recurstion defined in Equation 5.1.)

$f(0) = 1$
$f(1) = 1$
$f(i + 1) = P_2^2(i, \text{add}(f(i), f(\text{dec}(i))))$

2 Exercise 5.12 bls 164

Verify that iteration is primitive recursive. Afunction f is constructed from a function g by iteration if we have $f(x, y) = g^x(y)$, where we assume $g^0(y) = y$.

$f(0, y) = y$
$f(i + 1, y) = P_2^3(i, f(i, y), y)$

3 Exercise 5.13 bls 164

Verify that the function f defined as follows:

$$\begin{cases} f(0, x) = g(x) \\ f(i + 1, x) = f(i, h(x)) \end{cases}$$

is primitive recursive whenever g and h are.

$f(0, x, y) = g(P_1^2(x, y))$
$f(i + 1, x, y) = P_2^4(i, f(i, x, y), x, y)$
$y = h(x)$